

GCE

Mathematics

Advanced GCE

Unit **4735**: Probability and Statistics 4

Mark Scheme for June 2011

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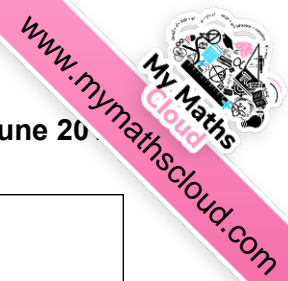
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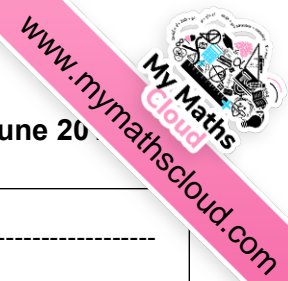
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<p>1 (i)</p>	$\sum_{x=0}^n \binom{n}{x} p^x q^{n-x} t^x$ $= \sum_{x=0}^n \binom{n}{x} (pt)^x q^{n-x}$ <hr/> <p>(ii)</p> $G_T(t) = (q+pt)^n (q+pt)^{2n}$ $= (q+pt)^{3n}$ <p>So $T \sim B(3n, p)$</p>	<p>M1</p> <p>A1</p> <p>2</p> <hr/> <p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>4</p> <p>[6]</p>	<p>From $E(t^x)$</p> <p>M1A0 \sum without limits $G_X(t)=q+pt$ M1 then argument A0</p> <hr/> <p>Multiplying pgfs</p> <p>For B For parameters</p>
<p>2 (i)</p>	<p>$H_0: m_d = 0, H_1: m_d > 0$, (where $d = \text{high} - \text{low}$)</p> <p>D: -4 3 6 1 12 7 14 16 11 -9 10</p> <p>Rank -3 2 4 1 9 5 10 11 8 -6 7</p> <p>$P = 57, Q = 9$</p> <p>$T = 9$</p> <p>$CV = 13$</p> <p>$9 < CV$ so reject H_0</p> <p>There is sufficient evidence at the 5% significance level to support the botanist's belief</p> <hr/> <p>(ii)</p> <p>The rank sum test is for independent samples, the H and L values are correlated</p>	<p>B1</p> <p>M1</p> <p>A1</p> <p>B1</p> <p>B1</p> <p>M1</p> <p>A1 ft</p> <p>7</p> <hr/> <p>B1</p> <p>1</p> <p>[8]</p>	<p>Or $H_0: m_H = m_L$, etc. Medians</p> <p>Ranking top down, -9,-10,8, ..M1A0 $T=15$ B0 [SR last 3 marks: $z=-2;09$ B1 <-1.96 etc M1A1] Or equivalent</p> <p>ft T</p> <hr/> <p>Accept data paired</p>
<p>3 (i)</p>	<p>$P(A B') = P(A \cap B') / P(B')$</p> <p>$\Rightarrow P(A \cap B') = 1/8$ AEF</p> <p>Use $P(A \cap B) = P(A) - P(A \cap B')$</p> <p>To give $P(A \cap B) = 5/8$ AEF</p> <hr/> <p>(ii)</p> <p>$P(A \cap B \cap C) = 5/8 \times 1/4 = 5/32$ AEF</p> <hr/> <p>(iii)</p> <p>$P(B \cap C) = 3\lambda/4$ and $P(C \cap A) = 3\lambda/4$</p> <p>Use formula for $P(A \cup B \cup C)$</p> <p>And $P(A \cup B \cup C) = 1$</p> <p>Sub into formula for $P(A \cup B \cup C)$ and solve for λ giving $\lambda = 3/16$ AEF</p>	<p>M1</p> <p>A1</p> <p>M1</p> <p>A1</p> <p>4</p> <hr/> <p>B1 $\sqrt{}$</p> <p>1</p> <hr/> <p>M1</p> <p>M1</p> <p>B1</p> <p>M1</p> <p>A1</p> <p>5</p> <p>[10]</p>	<p>May be implied</p> <p>Or equivalent</p> <hr/> <p>Ft 5/8</p> <hr/> <p>For use of both conditional probs Allow one sign error</p>
<p>4 (i)</p>	<p>$M'(t) = 3(1/4 + 3/4 e^t)^2 \times 3/4 e^t$</p> <p>$E(X) = M'(0) = 9/4$</p> <hr/> <p>(ii)</p> <p>mgf $(1/64 + 9/64 e^t + 27/64 e^{2t} + 27/64 e^{3t})$</p> <p>$P(X = 2) = \text{coefficient of } e^{2t} = 27/64$</p> <hr/> <p>(iii)</p> <p>Sum of 3 obs of Y with mgf $1/4 + 3/4 e^t$ has mgf of X</p> <p>$y: 0 \quad 1$</p> <p>$p: 1/4 \quad 3/4$</p> <p>$\text{Var}(Y) = 3/4 - (3/4)^2 = 3/16$</p>	<p>M1</p> <p>A1</p> <p>A1</p> <p>3</p> <hr/> <p>M1</p> <p>A1</p> <p>A1</p> <p>3</p> <hr/> <p>M1*dep</p> <p>A1</p> <p>*M1A1</p> <p>4</p> <p>[10]</p>	<p>Allow one error</p> <hr/> <p>Or $\text{PGF} = (1/4 + 3/4 z)^3$ expand find coefficient of z^2 $27/64$</p> <hr/> <p>OR $B(1, 3/4)$ Using $E(Y^2) - (E(Y))^2$ OR $1 \times 3/4 \times 1/4$ M0 if integration used</p>



5(i)	Does not require a known probability distribution	B1 1	Or equivalent
(ii)	$H_0: m_A = m_B, H_1: m_A \neq m_B$ Ranks: A 1 2 3 5 6 10 B 4 7 8 9 11 12 $R_A = 27, 78 - 27 = 51, \text{ so } W = 27$ OR: $R_B = 51, 78 - 51 = 27$ 5% CV = 26 $27 > CV$ so do not reject H_0 there is insufficient evidence at the 5% SL to indicate a difference in breaking strengths	B1 M1 M1 A1 B1 M1 A1 7	Medians Use N(39,39) with cc B1 $P(W \leq 27.5), Z = -1.84$ or equivalent M1 Not in CR etc A1
(iii)	B would have an extra rank 13 W still 27 but CV now 27 H_0 is now rejected	M1 B1 A1 3	$P(W \leq 27.5) = -2.07$ M1A1 In CR H_0 rejected A1
6(i)	$L=0, C=1$, choose 1C from 14 and 1 from 6 Others $14 \times 6 / {}^{36}C_2 = 2/15$ AG $L=1, C=1$, choose 1 from 16, 1 from 14 $16 \times 14 / {}^{36}C_2 = 16/45$ AG	M1 A1 M1 A1 4	Or ${}^{14}/_{36} \times {}^6/_35 \times 2$ Or ${}^{14}/_{36} \times {}^{16}/_{35} \times 2$
(ii)	Marginal C probs: 11/30 22/45 13/90 $E(C) = 22/45 + 26/90 = 35/45 = 7/9$	B1 M1 A1 3	AEF
(iii)	EITHER: $2 \times 1/42 + 2/15 + 16/105$ OR: $E(L) = 8/9, E(O) = 2 - 15/9 = 1/3$	M1 A1 A1 3	Other: 0 1 2 M1 p: ${}^{29}/_{42} \quad {}^2/_7 \quad {}^1/_42$ A1 $E(O) = {}^2/_7 + {}^2/_42 = {}^1/_3$ A1
(iv)	EITHER: Argument OR: Use idea that for independence $P(L \cap C) = P(L)P(C)$ Conclude that covariance is non-zero	B2 M1A1 B1 3	e.g The more Ls the fewer Cs OR Use conditional probability OR $\text{Cov}(L,C) = -136/405$ M1A1 L,C not indep B1
7(i)	$E(S) = \frac{1}{2}(E(\bar{U}_4) + E(\bar{U}_6))$ $= \frac{1}{2}(\mu + \mu) = \mu$, so S is unbiased $\text{Var}(S) = \frac{1}{4}(\sigma^2/4 + \sigma^2/6)$ $= 5\sigma^2/48$	M1 A1 M1 A1 4	With conclusion
(ii)	$E(T) = (a+b)\mu = \mu, a+b=1$ $\text{Var}(T) = a^2\sigma^2/4 + b^2\sigma^2/6$ Minimise $y = a^2/4 + b^2/6 = a^2/4 + (1-a)^2/6$ EITHER by differentiation OR, completing square, OR from a sketch graph. Giving $a = 2/5, b = 3/5$ Justify minimum value Variance = $\sigma^2/10$	M1 B1 M1 M1 A1 B1 A1 7	Allow from completion of square
(iii)	T is better since (both are unbiased and) $\text{Var}(T) < \text{Var}(S)$	B1 1	From calculated variances
(iv)	Sample mean of 10 observations (is also unbiased) with $\sigma^2/10$ They have the same efficiency	M1 A1 2	Or show that $T =$ mean of 10 observations
		[11]	
		[13]	
		[14]	

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